Short Papers

Investigation of Periodic Structures in a Fin Line: A Space-Spectral Domain Approach

N. Gupta and M. Singh

Abstract—**The space-spectral domain approach (SSDA) is utilized for the analysis of the unilateral fin line with periodically varying slot patterns. Two new type of structures in a unilateral fin line with sinusoidally and triangularly varying slot patterns are investigated using this approach.**

I. Introduction

Applications of periodic structures in slow wave devices and in filters are well known [1]–[3]. In microwave integrated circuits, these structures are usually formed by etching a periodic metallization on the circuit substrate. Generally, infinite periods are assumed in these structures and then Floquet's theorem is used to focus the problem into a single period. The spectral domain approach (SDA)[4] and the method of lines (MOL) [5], [6] have been used in the past to analyze various types of periodic structures in microstrip, fin line configurations, involving either 2-D expansion of basis functions for the SDA or 2-D discretization for the MOL. The SSDA [7], a novel combination of two methods, involves 1-D SDA in x-direction and 1-D MOL in the z-direction. This combination takes the advantage of the flexibility of the MOL in dealing with the arbitrarily shaped discontinuities and at the same time it utilizes the computational efficiency of the SDA.

In [8], the SSDA has been utilized to compute the resonant characteristics of the bilateral fin line resonators. The complexity arising due to the direct implementation of the matrices of large order has been minimized at the cost of some analytical preprocessing. This reduces the computation time, and at the same time, it also increases the efficiency of the existing algorithm significantly.

In the present paper, the above approach is extended to analyze a class of unilateral fin lines with slot pattern changing periodically in different fashion. In such cases, due to structural periodicity in the z-dimension, the periodic boundary conditions are required to be incorporated. The present analysis is simple and can be easily implemented on periodic slot pattern of various shapes, as long as the circuit contours can be described by a set of coordinates. The analysis establishes the functional behavior of the periodic fin line with respect to the propagation constant, depicting the passband and stopband phenomenon for the structures under consideration.

II. Theory

The discretization of resonators, periodic structures, and discontinuities exhibits a number of common features. However, the structures differ in the boundary conditions and hence their *difference matrices* and the *transformation matrices* are different.

The structure considered is a unilateral fin line as shown in Fig. 1(a). For the periodic structures, the potential functions and all electromagnetic field components must satisfy Floquet's theorem, as

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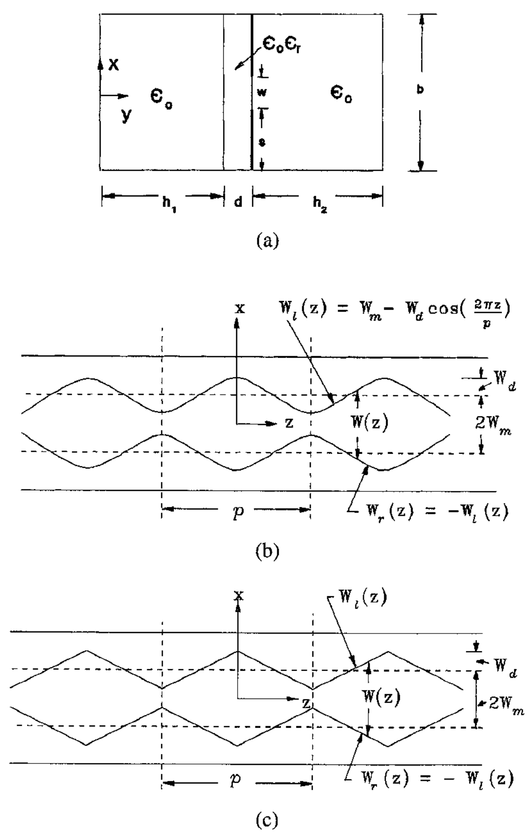

Fig. 1. (a) Cross-sectional view of unilateral fin line. (b) Sinusoidal slot variation. (c) Triangular slot variation.

$$\psi^{e,h}(x, y, z+p) = e^{-j\beta p}\psi^{e,h}(x, y, z) \tag{1}$$

where β is the propagation constant in the z-direction and p is the periodic length.

One period of the structure is discretized with ψ^e lines located at $z = kh_z$, and ψ^h lines at $z = (k+0.5)h_z$ $(k = 1, ..., N)$, where h_z is the discretization interval. The finite difference expression for the first derivative of ψ^e with respect to z is then given by

$$h_z \frac{\partial \psi^e}{\partial z} \rightarrow [\psi^e][D_z]^t \tag{2}$$

with

$$[D_z] = \begin{bmatrix} -1 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ e^{-j\beta p} & & & -1 \end{bmatrix}. \tag{3}$$

The difference operator for ψ^h and its derivative is $-[D_z]^{*t}$, which yields for the second-order derivative of ψ^e. Therefore

$$h_z^2 \frac{\partial^2 \psi^e}{\partial z^2} \rightarrow -[\psi^e][D_z]^t[D_z]^* = [\psi^e][D_{zz}^e]^t. \tag{4}$$

The original form of the partial Helmholtz equation, after application of the SDA in x-direction and the MOL in z-direction reduces to

$$\frac{d^2\psi^{e,h}}{dy^2} - \left(\frac{[D_{zz}^{e,h}]}{h_z^2} + (\alpha_n^2 - k_i^2)I\right)\psi_i^{e,h} = 0 . \quad (5)$$

The matrix $[D_{zz}]$ is a Hermitian matrix and is to be transformed into a diagonal matrix of eigenvalues $[\delta]$ using unitary transformation matrices of eigenvectors. For periodic boundary conditions, the eigenvalues and eigenvectors matrices are derived as in [9].

The uncoupled differential equations obtained in the transformed domain are

$$\left(\frac{d^2}{dy^2} - \gamma_i^{e,h}\right)\vec{\phi}_i^{e,h}(\alpha_n, y) = 0 \quad (6)$$

where

$$\gamma_i^{e,h} = \left(\frac{[\delta]^2}{h_z^2} + \alpha_n^2 - k_i^2\right)^{\frac{1}{2}} \quad (7)$$

and

$$\vec{\phi}_i^{e,h} = [T^{e,h}]\psi_i^{e,h} \ . \quad (8)$$

The solution of (6) describes the wave propagation in y direction and establishes a relationship between continuity conditions at the two boundaries of the dielectric interface.

The space-spectral coupled admittance matrix which is obtained by application of the continuity condition in the plane where the slot pattern is situated and introducing the boundary conditions at the metallic covers at $y = 0$ and $y = h_1 + h_2 + d$ is

$$\begin{bmatrix}\tilde{J}_x(\alpha_n)\\ \tilde{J}_z(\alpha_n)\end{bmatrix} = \begin{bmatrix}[\tilde{G}_{11}(\alpha_n, k_0, \beta)] & [\tilde{G}_{12}(\alpha_n, k_0, \beta)]\\ [\tilde{G}_{21}(\alpha_n, k_0, \beta)] & [\tilde{G}_{22}(\alpha_n, k_0, \beta)]\end{bmatrix}\begin{bmatrix}\tilde{e}_z(\alpha_n)\\ \tilde{e}_x(\alpha_n)\end{bmatrix}. \quad (9)$$

A reverse transformation is then carried out for both MOL and SDA independently since the final boundary conditions cannot be applied in the transformed domain. For the MOL, the back transformation is done with the help of the orthogonal transformation matrices, and for the SDA it is done by applying the Galerkin's technique. In the case of resonators, the Galerkin's technique is applied after obtaining the reduced matrix by considering few discretization lines passing through the resonator portion, while in the case of periodic structures with continuous slot patterns, the operations are performed considering all the lines within a period.

After reverse transformation, the final boundary conditions lead to the Hermitian matrix, which is then solved for zero of the determinant.

In order to achieve a fast algorithm for the periodic slotting centered in the E-plane, 1-D sinusoidal basis functions modified by an edge-condition term [8] are generally employed. For a periodically varying slot width, the parameter w and s, as in Fig 1(a), are the function of z. Hence, they are different for each electric and magnetic lines and can be easily described by a set of coordinates for w and s both.

III. Results

The convergence behavior of the numerical method has been discussed for the example of a periodically nonuniform unilateral fin line with slot width changing in sinusoidal and triangular fashion. The substrate material used is Duroid ($\epsilon_r = 2.22$) with a thickness, $d = 0.127$ mm. The geometrical dimensions of the slot structure are as shown in Fig. 1(b) and (c), with periodicity (length of one period) of the line $p = 3.048$ mm. The metallization thickness is assumed to be zero. The periodically varying width of the slot for the sinusoidal and triangular variation is defined by the two periodic functions $W_l(z)$ (left side) and $W_r(z)$ (right side) as a function of the z-coordinate. The width of the slot at the coordinate z is therefore $W(z) = W_l(z) - W_r(z)$.

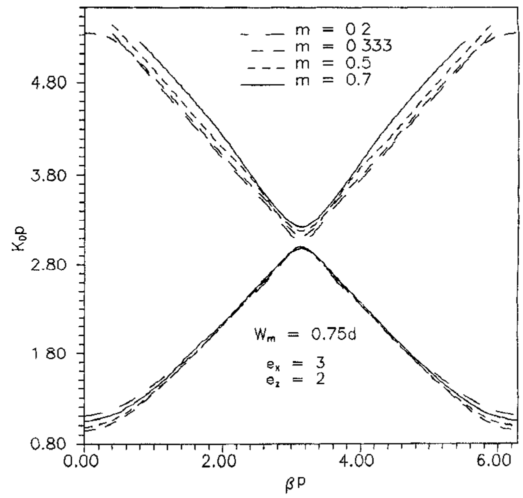

Fig. 2. Dispersion characteristic for sinusoidal slot variation.

The stability of the calculated normalized β in relation to the different frequencies, i.e., the dispersion characteristics in the $k_0 - \beta$ plane, commonly known as the Brillouin diagram, has been taken as a criteria for the convergence of the numerical method. It mainly depends on the number of discretization lines within a period, the chosen number of expansion functions, as well as on the number of spectral terms. Additionally, the computation results are influenced by the accuracy with which the expansion functions are described in the spectral domain. Our experience says that the sinusoidal basis functions modified by an edge condition term can be employed for most of the slot geometry. Furthermore, the computations show that the convergence is also frequency dependent. In view of the required accuracy in the practical applications, the discretization interval is chosen to be at least one-sixth of the guiding wavelength [11].

The validity of the present analysis is first checked for the uniform slot pattern case. Next, an attempt is made to study periodic structures in a fin line for which no previous data was available, like sinusoidally and triangularly varying slot patterns. Fig. 2 shows the dispersion characteristics for the sinusoidal slot variation case with 300 spectral terms, 15 number of lines in a period and total five number of basis functions ($e_x = 3, e_z = 2$), for the four sets of data for the modulation index as m = 0.2, 0.333, 0.5, 0.7. The stopband changes with the modulation index. As the modulation index is increased, the stopband also increases. For larger values of modulation index, such as m = 0.7, total five number of basis functions are not sufficient to reach convergence; hence, more than five number of basis functions have to be taken into consideration. It is also observed that the convergence is better in the higher-frequency region than in the lower-frequency region. The improvement of the convergence with increasing frequency can be explained easily. For the lower-frequency region, one wavelength is equal to several periodic lengths p of the line, while in the higher-frequency range, the periodic length is equal to or smaller than the wavelength of the wave on the line. This means that the error per wavelength is larger for low frequencies and smaller for high frequencies.

Fig. 3 shows the dispersion characteristics for the sinusoidally varying slot pattern when W_m and W_d, both change for each set of parameters.

Fig. 4 shows a comparison in the dispersion characteristics for the sinusoidally and triangularly varying slot patterns. The stopband for the triangularly varying slot pattern is marginally narrower than the

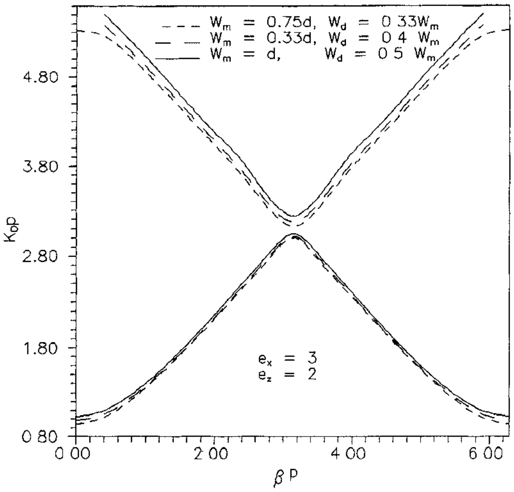

Fig. 3. Dispersion characteristic for sinusoidal slot variation.

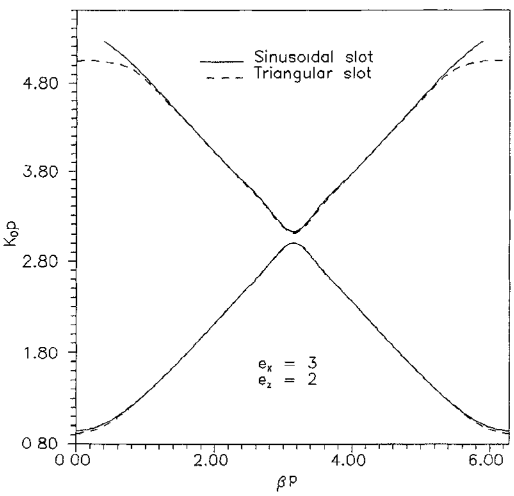

Fig. 4. Dispersion characteristics for sinusoidal and triangular slot variation.

stopband of the sinusoidally varying slot pattern. This may be due to broader apex of the sinusoidal slot in comparison to sharper apex of the triangular slot.

IV. Conclusion

The SSDA has been extended for computing the dispersion characteristics of some periodic structures in the $k_0 - \beta$ plane. With the incorporation of the periodic boundary conditions, the present method is very well suited to analyze various possible periodic structures in microstrip, fin lines, and co-planar waveguides, which were difficult to analyze before. The interesting feature of the method is that the same set of the sinusoidal basis functions can be utilized for most of slot geometry. This method can also be utilized to study other types of periodic structures, like meander lines and so on.

References

[1] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960.

[2] F. J. Glandrof and I. Wolf, "A spectral-domain analysis of periodically nonuniform microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 35, pp. 336–343, Feb. 1987.

[3] ——, "A spectral-domain analysis of periodically nonuniform coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 32, pp. 684–688, Feb. 1988.

[4] T. Kitazawa and R. Mittra, "An investigation of striplines and finlines with periodic stubs," *IEEE Trans. Microwave Theory Tech.*, vol. 32, pp. 684–688, 1984.

[5] S. B. Worm and R. Pregla, "Hybrid-mode analysis of arbitrarily shaped planar microwave structure by the method of lines," *IEEE Trans. Microwave Theory Tech.*, vol. 32, no. 2, 1984.

[6] W. Pascher and R. Pregla, "Full wave analysis of complex planar microwave structures," *Radio Sci.*, vol. 22, no. 6, pp. 999–1002, Nov. 1987.

[7] K. Wu and R. Vahldieck, "A new method for the modeling three-dimensional MIC/MMIC circuits: A space-spectral domain approach," *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 9, pp. 1309–1318, Sept. 1990.

[8] N. Gupta and M. Singh, "The space spectral domain technique applied to a fin line configuration," *IEEE Microwave and Guided Wave Lett.*, vol. 3, no. 5, pp 125–126, 1993.

[9] R. Pregla and W. Pascher, "The method of lines," in *Numerical Techniques for Microwave and Millimeter Wave Passive Structures*, T. Itoh, Ed. New York: Wiley, 1989, ch. 6, pp. 381–446.

[10] B. Bhat and S. K. Koul, *Analysis, Design and Application of Fin Lines*. Norwood, MA: Artech House, 1987.

[11] K. Wu, M. Yu, and R. Vahldieck, "Rigorous analysis of 3-D planar circuit discontinuities using the space-spectral domain approach (SSDA)," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1475–1483, July 1992.

Complex Solitons in a Superconductive Medium

K. Hayata and M. Koshiba

***Abstract*—We show analytically that a type-II superconductor may support short-range electromagnetic spatial solitons with a complex propagation constant. A theoretical model based on the Ginzburg-Landau theory is used. Analytical results for the complex solitons predict unique features that cannot be found in conventional solitons in normal (asuperconductive) media.**

I. Introduction

Soliton and solitary-wave propagation in material media such as dielectrics, semiconductors, plasmas, and magnetized materials have long been of extensive interest in a rich variety of branches that include both pure and applied sciences [1]. For solitary light beams (spatial solitons) that are describable with a family of nonlinear Schrödinger equations, a picture that explains solitons in terms of the fundamental modes of the linear waveguide they induce was found to be consistent with our physical intuition [2], [3]. As is well known in classical waveguide theory, guided modes in a linear waveguide can be classified into three types: bound (oscillatory), evanescent (diffusive), and complex modes, which can be characterized by a real, a purely imaginary, and a complex propagation constant, respectively.

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